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APPLICATION OF A PRIORI INFORMATION TO ASSURE IDENTIFIABILITY OF A MATHEMATICAL MODEL

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The possibility of simultaneous determination of the heat elimination coefficient and the temperature of the environment as a function of two variables that are in the model in the same dimensionality as the desired quantities is studied.

To a significant extent the development of the theory of inverse problems broadens the data processing methodology. In particular, a study of questions of the identifiability of mathematical models displays the possibility of simultaneously finding a whole series of parameters of the object by observing its unique state function [1]. A computational-experimental determination of several unknown object characteristics at one time, including those that are difficult of access by direct measurements, permits realization of an interpretation of the results of a complex experiment in which the observations are performed by traditional control-measurement facilities. Consequently, it turns out to be possible to pose and solve a question on raising the informativity of observations with significant complication of the technical support of the experiment.

Planning and executing appropriate experiments should assure conservation of conditions to retain the mutually one-to-one correspondence between the desired quantities and the observable values of the object state. The method proposed in [2] can be utilized to investigate the mutually one-to-one correspondence between the coefficients of a mathematical model and its state that is considered given at each point of the domain of variation of the independent variables. The approach being developed permits answering a number of important practical questions on the identifiability of mathematical models by indicating in advance the class of ambiguity of the solution of the problem under consideration as well as obtaining simple criteria and conditions for conserving the mutually one-to-one correspondence needed for a preliminary estimation of the possibility of identifiability of the object under consideration.

By using this method, an analysis of heat-transfer processes was performed in [3], which answered the question of the possibility of simultaneous determination of both the heat elimination coefficient and the temperature of the environment by means of observation of the temperature field within the test object. One of the main results of this paper is the deduction that if finding the heat elimination coefficient and the temperature of the environment is not allowed under general assumptions about the properties of the desired quantities, then

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their joint identification turns out to be possible if the form of the functional dependences by which the unknown parameters are approximated is narrowed in advance, in conformity with a given model.

Narrowing the form of the functional dependences is used extensively to assure the uniqueness of the solution of inverse problems [4, 5]. The narrowing methods being examined in these and analogous papers are based on diminishing the number of independent variables in the desired quantities. The purpose of the present paper is to study the approach within whose framework identifiability of the mathematical model is conserved successively during rejection of the condition of reducing the numbers of independent variables in the desired quantities. The requirement of definite parametrization of the functions wherein a priori information about the object properties are reflected, is at the bottom of the approach being proposed. Consequently, the build up of the identifiability of a given model reduces to the selection of those desired functions that would, on one hand, satisfactorily approximate the unknown quantities, and on the other, would conserve the requisite mutually one-to-one correspondence between the state of the object and the desired parameters.

Let us examine the practical possibilities of the approach being proposed for a two-dimensional object whose description includes unknown functions of two variables. Let us take the problem continuing the investigation started in [3] as a model example, and let us select the object to which considerable attention is attracted in applied cryogenics [6].

Let a thermal system that is a reservoir filled with cryogenic fluid be considered given. The reservoir outer surface is considered heat-insulated while homogeneity of the temperature field is assumed at one of the boundary points, say, because of object symmetry.

In the general case mathematical modelling of the described object is expressed by a complex system of differential equations [7]. Significant simplification of the kind of models being utilized can be achieved if the heat transfer conditions between the reservoir wall and the cryogenic agent are considered known. Experimental investigations of such systems permit sampling of observations of the temperature field of the reservoir wall to be obtained. Then from an appropriate adjoint formulation [7], the heat conduction problem in the wall can be deduced as an independent model under the condition that processes within the reservoir are replaced by equivalent thermal relations in the form of variable heat transfer conditions to be determined later. In this connection, we study the question of the possibility of simultaneous determination of the wall heat elimination coefficient and the temperature of the cryogenic fluid by using the following mathematical model

$$
\begin{gathered}
c(u)\,\rho\,\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left[\lambda(u)\,\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda(u)\,\frac{\partial u}{\partial y}\right],\quad x_0<x<x_k,\\
y_0<y<y_k,\ t>0;\\
u|_{t=0}=u_0(x,\ y),\quad x_0<x<x_k,\quad y_0<y<y_k;\\
\left.\frac{\partial u}{\partial y}\right|_{y=y_0}=0,\quad u|_{y=y_k}=v(x,\ t),\quad x_0<x<x_k,\ t>0;\quad \alpha(y,\ t)\,[u|_{x=x_0}-\\
-u_{\text{cp}}(y,\ t)]-\lambda\left.\frac{\partial u}{\partial x}\right|_{x=x_0}=0,\quad y_0<y<y_k,\ t>0;\quad \left.\frac{\partial u}{\partial x}\right|_{x=x_k}=0,\\
y_0<y<y_k,\quad t>0.
\end{gathered}
\tag{1}
$$

Under general assumptions about the form of the desired functions $\alpha(y, t)$ and $u_{av}(y, t)$, the problem of interest to us has no unique solution since a value can always be indicated

$$\alpha''=\alpha'\,\frac{u|_{x=x_0}-u'_{av}}{u|_{x=x_0}-u''_{av}},$$

that will conserve the solution of problem (1) unchanged for arbitrary variations of α', u_{av}', and u_{av}''.

Taking into account that an essential singularity of filling the reservoirs with cryogenic fluids is the jumplike change in the heat transfer conditions during passage through the phase interfacial boundary, we represent the desired quantities in the form

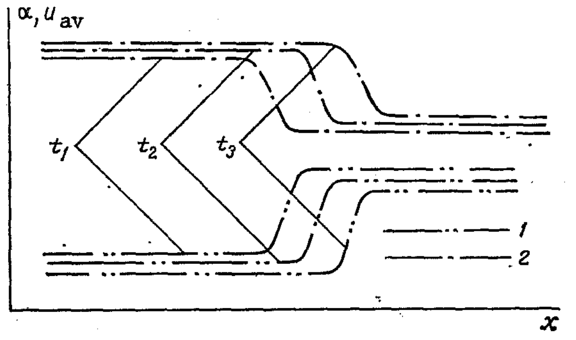

Fig. 1. Nature of the change in heat-transfer conditions during filling a reservoir with cryogenic fluid at different times $t_1 < t_2 < t_3$: 1) heat elimination coefficient, 2) temperature of the environment.

$$\alpha(y,\ t) = A_1 \operatorname{arctg}(A_2 y + A_3) + A_4, \tag{2}$$

$$u_{av}(y,\ t) = B_1 \operatorname{arctg}(B_2 y + B_3) + B_4, \tag{3}$$

where $A_i(t)$, $B_i(t)$ are arbitrary functions to be determined.

The selected approximation corresponds to the nature of the change in the heat transfer in the reservoir as it is filled with cryogenic fluid: heat transfer is realized from the gas phase up to the time the phase interfacial boundary reaches a given point and afterwards interaction with the liquid phase sets in. Such a nature of the change in heat transfer conditions is shown in the figure. Similar jumplike processes are approximated satisfactorily by functions of the type (2) and (3).

Let us note that the parametrization (2) and (3) corresponds to the question of identifiability of the heat-transfer conditions under consideration. The final answer on its sufficiency can be obtained during a further study of the mutually one-to-one correspondence between the desired functions and a discrete set of observations.

We investigate the question of conserving the mutually one-to-one correspondence between the temperature field that is described by the model (1) and functions of the form (2) and (3) by the method of contradiction by considering that a transformation of the coefficients

$$A_i''(t) = A_i'(t) + a_i(t),\quad B_i''(t) = B_i'(t) + b_i(t),$$

exists that does not result in a change in the temperature field $u_*(x,\ y,\ t)$.

From the condition of conservation of the heat flux on the boundary $x = x_0$

$$\alpha'(u_*|_{x=x_0} - u_{av}') = \alpha''(u_*|_{x=x_0} - u_{av}'')$$

we obtain

$$\begin{aligned}
&[A_1' \operatorname{arctg}(A_2' y + A_3') + A_4'][B_1' \operatorname{arctg}(B_2' y + B_3') + B_4'] - \\
&-[(A_1' + a_1)\operatorname{arctg}(A_2' y + A_3' + a_2 y + a_3) + A_4' + a_4] \times \\
&\times [(B_1' + b_1)\operatorname{arctg}(B_2' y + B_3' + b_2 y + b_3) + B_4' + b_4] = \\
&= u_*|_{x=x_0}[A_1' \operatorname{arctg}(A_2' y + A_3') + A_4' - \\
&-(A_1' + a_1)\operatorname{arctg}(A_2' y + A_3' + a_2 y + a_3) - A_4' - a_4].
\end{aligned}$$

By expanding the last expression a general functional form of the assumed nonidentifiable temperature field can be found

$$u_*(x,\ y,\ t) = V(x)\,W(t)\operatorname{arctg}[F(t)\,y + G(t)] + H(t), \tag{4}$$

where V, W, and F are functions different from zero.

Let us calculate the derived function (4) with respect to the coordinate y and let us equate it to zero at the point $y = y_0$ according to the given boundary condition. We consequently obtain the function F identically zero which does not agree with the dimensionality of the state under consideration. Therefore, the assumption made about the existence of an invariant transformation of the coefficients A_i and B_i contradicts the properties of the solution of the problem (1) upon giving coefficients of the form (2) and (3). In its turn this indicates conservation of the mutually one-to-one correspondence between heat-transfer conditions of the form (2) and (3) and the temperature field that is described by the model (1). Which was to have been proved. (Q.E.D.)

We now examine the question of whether parametrization of the heat transfer conditions in the form (2) and (3) is sufficient for identifiability of the thermal system being studied under more general assumptions about its properties. We get rid of the symmetry condition utilized above at the point $y = y_0$ and we consider the value $v_0(x, t)$ of the temperature to be given at a certain point $y = y_0$ without any constraints on its form.

We substitute the state found (4) into the original heat conduction equation. After the necessary manipulations, conditions can be found which the functions V, W, F, and G from (4) should satisfy. In particular, we have for the last two

$$c\rho\left(\frac{dF}{dt}y+\frac{dG}{dt}\right)=\frac{\partial\lambda}{\partial y}F-\lambda\frac{2F^2(Fy+G)}{1+(Fy+G)^2}.$$

Expanding the expression obtained in powers of y we also find that $F \equiv 0$. It hence follows that observations on the state of an object at an arbitrary segment of the reservoir length are identifiable as a whole relative to the desired heat transfer conditions.

Summarizing the results obtained, the following deductions can be made. An approach is proposed that permits rejection of diminution of the number of independent variables in the desired quantities and their conservation in conformity with the dimensionality of the given model during formulation of the identification problem. It is shown that under conditions when the general assumptions result in ambiguity of the selection of the desired quantities, involvement of a priori information about the physical features of the process in progress in the form of a certain parametrization can assure conservation of a mutually one-to-one correspondence between the object state and its desired parameters. The form found for representation of the heat transfer conditions in a cryogenic reservoir permits a subsequent correct formulation of the identification problem for the object considered. In conclusion, we note that the approach elucidated is not limited to the case of just two-dimensional models. A one-dimensional formulation, say, can be studied analogously to the investigation performed above, and any other functional representation of heat transfer conditions can also be considered.

NOTATION

u, temperature field; u_0, initial distribution; v, v_0, boundary temperatures; c, specific heat; ρ, density; λ, heat conductivity; α, heat elimination coefficient; u_{av}, temperature of the environment; x_0, x_k, y_0, y_k, spatial boundaries of the object; A_i, B_i, functions defining parametrization of the desired quantities; V, W, F, G, H, functions defining the functional representation of the nonidentifiable temperature field.

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